

Lecture 6: Using Shocks to Identify Structural Equations

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- Last few lectures: **semi-structural time series methods** to identify macro shocks
- We can now reap the rewards & return to our **substantive questions**:
 1. How can we estimate “**structural**” macroeconomic equations?
 2. How should we optimally design **business-cycle stabilization policy**?
 3. What are the sources of **business-cycle fluctuations**?
- This lecture: **estimating structural relationships**

Today's lecture

- Important objective of empirical macro is to learn about certain aggregate “structural” relationships predicted by theory. Some notable examples:

- **NKPC**

$$\pi_t = \lambda y_t + \beta \mathbb{E}_t [\pi_{t+1}] + \text{shocks} \quad (1)$$

- **Euler equation/EIS**

$$c_t = \mathbb{E}_t [c_{t+1}] - \frac{1}{\gamma} \mathbb{E}_t [i_t - \pi_{t+1}] + \text{shocks} \quad (2)$$

- How should we estimate (1) and (2)? We'll review a couple of strategies today:
 1. Classical approach: single-equation estimation via **lagged instruments**
 2. Recent time series alternative: project on identified **structural shocks**
 3. Very brief preview: **cross-sectional** (cross-individual, cross-regional, ...) variation

Outline

1. Single-Equation Estimation Through Lagged Instruments

NKPC

Elasticity of intertemporal substitution

2. Regressions in Impulse Response Space

General Idea

Application: NKPC

3. Cross-Sectional Strategies

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Challenges for NKPC estimation

- Return to the NKPC, just with slightly more general notation:

$$\pi_t = \lambda y_t + \gamma_f \mathbb{E}_t [\pi_{t+1}] + \text{shocks} \quad (3)$$

- What are the hurdles to estimating (3)?
 1. It is hard to measure the output gap y_t
Also: theory only robustly predicts that marginal costs should appear in (3). The output gap showing up here requires additional assumptions. Will ignore that for today.
 2. It is hard to measure inflation expectations
 3. We don't know what kind of (supply) shocks hit (3), and thus can't measure them. But we can't ignore them, since they are likely correlated with right-hand side variables
- **Q:** how can we hope to estimate (3) in light of these challenges?

A simple approach to NKPC estimation

- We'll first review an approach popular in the late-1990s/early 2000s. Mostly focus on **Galí and Gertler (1999)** as the most well-known example.
- Main idea: **three simplifying assumptions** [+ rational expectations]
 1. We are correctly able to measure the **forcing variable** y_t
Whether it's marginal cost, unemployment, output gap, ...
 2. Inflation expectations are **rational**
 3. Either there are **no supply shocks**, or supply shocks are fully **transitory** (iid ε_t^s)
- Under those assumptions we have

$$\pi_t = \lambda y_t + \gamma_f \pi_{t+1} + \underbrace{\varepsilon_t^s + \gamma_f \{ \mathbb{E}_t [\pi_{t+1}] - \pi_{t+1} \}}_{\text{error term}} \quad (4)$$

Note: error term is uncorrelated with info at t (if no supply shocks) or $t - 1$ (if static supply shocks).

A simple approach to NKPC estimation

- This suggests the following simple IV estimation strategy: orthogonality conditions

$$\mathbb{E}_t \{(\pi_t - \gamma_f \pi_{t+1} - \lambda y_t) z_t\} = 0$$

where the instruments are macro observables dated at or usually before time t

- In [Galí and Gertler \(1999\)](#): use four lags of inflation, labor share, output gap, long-short interest rate spread, wage inflation, commodity inflation
 - Find sensible results. Extended (better-fitting) specifications also allow for some inflation inertia [theory: infl. indexing]. Estimation proceeds through similar IV strategies.
- **But:** subsequent work cast doubt on those findings
 - Results are highly sensitive to instruments used, vintage of data, changes in model specification, ... [see [Mavroeidis, Plagborg-Møller and Stock \(2014\)](#)]
 - Why? inflation is hard to forecast → weak instruments

A simple approach to NKPC estimation

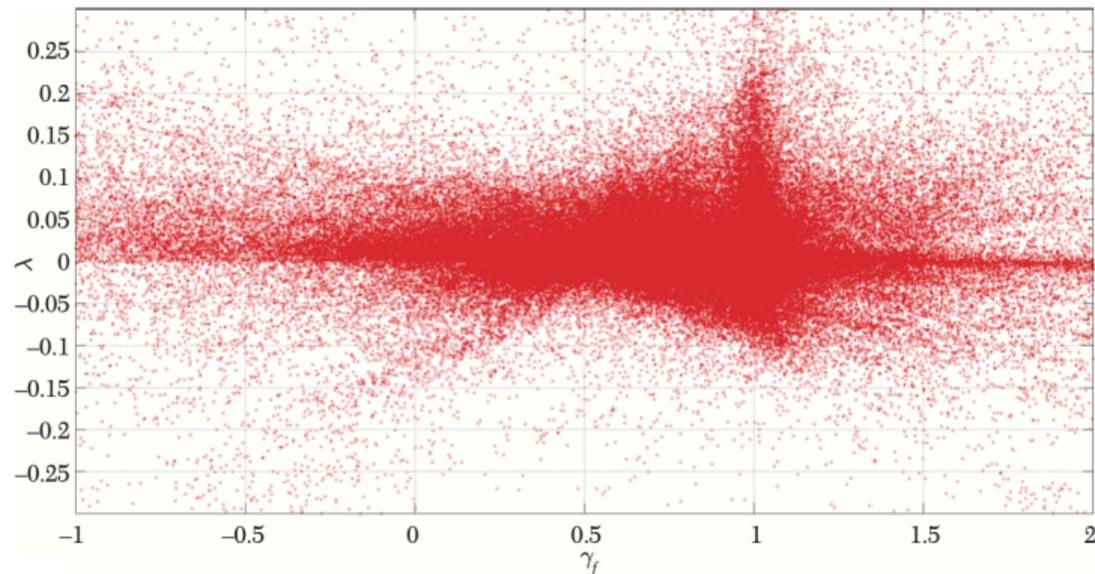


Figure 5. Point Estimates: Output Gap Specifications

Notes: Point estimates of λ , γ_f from the various specifications listed in table 4 that use the output gap as forcing variable, excluding real-time and survey instrument sets.

Source: Mavroeidis, Plagborg-Møller and Stock (2014)

Takeaways

- **Mavroeidis et al.** arrive at a pessimistic conclusion:

The literature has reached a limit on how much can be learned about the New Keynesian Phillips curve from aggregate macroeconomic time series. [...] New identification approaches and new datasets are needed to reach an empirical consensus.

- We will consider both:
 1. New identification approaches using aggregate time series data
 2. Alternative cross-sectional approaches [**briefly today, more later**]

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Challenges for EIS estimation

- Very similar issues to the NKPC literature, so my discussion will be brief
- We want to estimate:

$$c_t = \mathbb{E}_t [c_{t+1}] - \frac{1}{\gamma} \mathbb{E}_t [r_{t+1}] + \text{shocks} \quad (5)$$

- We face similar hurdles as before:
 1. It is hard to measure the forcing variable r_t
Plus note that here we need expectations of the forcing variable, which is even harder.
 2. It is hard to measure consumption expectations
 3. We don't know what kind of (demand) shocks hit (5), and thus can't measure them. But we can't ignore them, since they are likely correlated with right-hand side variables
- Standard solution is again a **lagged aggregate instruments** strategy

A simple approach to EIS estimation

- Canonical contributions to this literature are [Hall \(1989\)](#), [Campbell and Mankiw \(1989\)](#) and [Hansen and Singleton \(1983\)](#)
- Main idea: **three simplifying assumptions** [+ rational expectations]
 1. We are correctly able to measure the **forcing variable** r_t
 2. Expectations are **rational**
 3. Either there are **no demand shocks**, or demand shocks are fully **transitory** (iid ε_t^d)
- Under those assumptions we have

$$c_t = c_{t+1} - \frac{1}{\gamma} r_{t+1} + \underbrace{\varepsilon_t^d + \mathbb{E}_t [c_{t+1}] - c_{t+1} - \frac{1}{\gamma} \{ \mathbb{E}_t [r_{t+1}] + r_{t+1} \}}_{\text{error term}} \quad (6)$$

Note: error term is uncorrelated with info at t (if no demand shocks) or $t - 1$ (if static demand shocks).

Results & discussion

- Results are similarly erratic across specifications, thus no consensus on γ
- More generally: the entire approach here relies heavily on

$$c_t = \mathbb{E}_t [c_{t+1}] - \frac{1}{\gamma} \mathbb{E}_t [r_{t+1}] + \text{shocks} \quad (7)$$

being a **correctly specified structural equation**. What about:

- **Multiple assets?** Binding **liquidity constraints**?
- **Behavioral frictions** in household behavior? Non-rational expectations?
- **Adjustment costs** for durable consumption?
- **Non-separable utility** over consumption and labor supply?

In those cases we would have a much more general agg. consumption function, e.g.

$$\mathbf{c} = \mathcal{C}(\mathbf{y}, \mathbf{r})$$

Of course there are similar concerns for the NKPC. But simple IS curves seem even more at odds with macro data than NKPC-like relationships ...

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Identifying macro equations through structural shocks

- An alternative idea was proposed by [Barnichon and Mesters \(2020\)](#). I will briefly review the core logic & their main application (to NKPC estimation)
- The idea is really simple: **IRFs to macro shocks** can trace out structural equations
 - Let's suppose we've identified a monetary policy shock ε_t^m , and let IRFs to that shock be indicated by m superscripts
 - If there's actually a standard NKPC in the economy, then the IRFs satisfy

$$\pi_h^m = \lambda y_h^m + \gamma_f \pi_{h+1}^m, \quad h = 0, 1, \dots$$

Relies on the functional form of the NKPC, but can accommodate very general (e.g., persistent) supply shocks, & works for all monetary shocks. Key is that (i) IRFs to MP shocks are conditional expectations & (ii) MP does not affect the supply shocks—it moves us along the NKPC.

- Similarly, if there's a standard Euler equation, then the IRFs satisfy

$$c_h^m = c_{h+1}^m - \frac{1}{\gamma} r_{h+1}^m, \quad h = 0, 1, \dots$$

Implementation details

- This suggests that we can get structural coefficients like $\{\lambda, \gamma\}$ through “regressions in impulse response space”
 - *Unconditional variation* in macro time series does not trace out structural equations, and using lags as IVs is not particularly promising
 - But *conditional variation*—induced by well-chosen identified macro shocks—may well trace out such structural relations
- **Econometric implementation** [see Lewis-Mertens (2022) for a recent, improved approach]

- The actual implementation is simple—use current and lagged shocks as instruments for the macro equation, so e.g.

$$\mathbb{E}_t \{(\pi_t - \gamma_f \pi_{t+1} - \lambda y_t) z_t\} = 0$$

where now $z_t = (\varepsilon_t^m, \varepsilon_{t-1}^m, \varepsilon_{t-2}^m, \dots)'$

- Some words of caution: # of lags matters, many instrument issues, results are a bit sensitive to their specific way of collapsing the number of instruments, ...

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Application: NKPC estimation

- **Barnichon and Mesters** wish to estimate the following NKPC specification:

$$\pi_t = \gamma_b \pi_{t-1}^4 + \gamma_f \mathbb{E}_t [\pi_{t+4}^4] + \lambda y_t + \varepsilon_t^s \quad (8)$$

- Here a “4” superscript denotes averaged inflation over the past year
- Will furthermore impose that $\gamma_f + \gamma_b = 1$
- **Identification**
 - Estimate (8) using current and lagged monetary policy shocks [from Gertler-Karadi] as instruments
 - For forcing variable consider both unemployment as well as a measure of the output gap
 - Paper also has results using other shocks [Romer-Romer] and with $\gamma_b + \gamma_f \neq 1$

Results

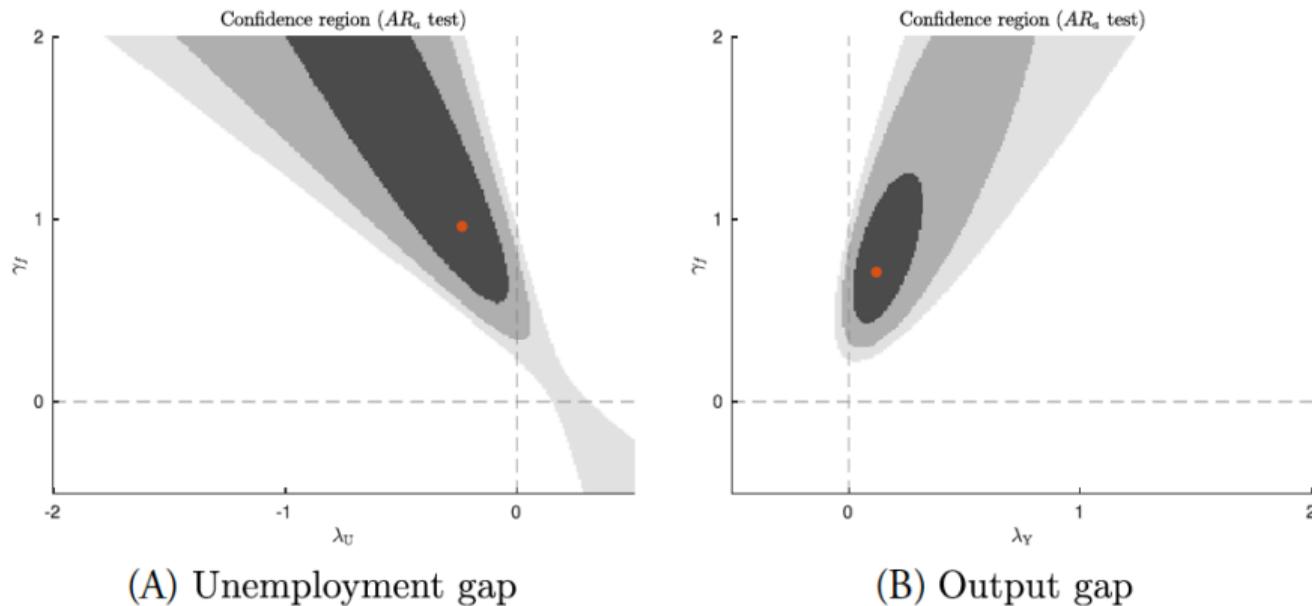


FIGURE V

The Phillips Curve, 1990–2017, HFI id, $\gamma_f + \gamma_b = 1$

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Cross-sectional strategies

- The pure **time series approaches** we've seen so far have some limits:
 1. **Lagged aggregate instruments**: many & weak instruments, dubious exclusion restrictions
 2. **Macro shocks**: more credible, but still requires strong functional form restrictions and is quite fiddly in practice, also many & weak IV issues
- How about **cross-sectional strategies**?
 - Later in this class we'll have dedicated lectures about this, so just a very quick preview now
 - Our overall takeaway in those future lectures will be that micro data are well-suited to **identify individual model blocks** = individual equations [rather than the full Θ 's that we usually want in our impulse-propagation paradigm ...]
 - This seems ideal for the purposes of this lecture! But there are some important subtleties. Next two slides will preview the promise but also the subtleties using the Euler equation and NKPC examples

Cross-sectional data and consumption functions

- Go beyond a simple Euler equation to a general **aggregate consumption function**:
Recall from the problem set how this strictly generalizes the usual Euler equation ...

$$\mathbf{c} = \mathcal{C}(\mathbf{y}, r) \Rightarrow \hat{\mathbf{c}} = \mathcal{C}_y \times \hat{\mathbf{y}} + \mathcal{C}_r \times \hat{r} \quad (9)$$

- \mathcal{C}_y : matrix of intertemporal MPCs
- \mathcal{C}_r : response to interest rates, will combine EIS and any potential wealth effects
- What we will see later: “ideal” X-sectional experiments identify (parts of) \mathcal{C}_y and \mathcal{C}_r
 - We will work this out explicitly in class for \mathcal{C}_y
 - Somewhat trickier for interest rates and so EIS. One nice example is [Gruber \(2013\)](#).

X-sectional approaches are more promising to learn about (9) than time-series IS curve estimation

Cross-sectional data and NKPCs

- Things are a lot more complicated for the NKPC—it's no simple **firm supply equation!**
 - Recall the textbook derivation: starts with a firm problem, but then leverages symmetry across firms, rational expectations, household labor supply decisions, ...
 - This means: we can't just look at pricing decisions of individual firms to learn about an aggregate NKPC—if such a relation exists then it necessarily embeds some GE!
- **Natural candidate:** regional variation. Maybe less endogeneity issues than aggregate time series, but still enough GE to be informative about NKPC slope?
 - An important contribution that develops and then uses this (here admittedly very vague!) intuition is [Hazell, Herreño, Nakamura and Steinsson \(2021\)](#)
 - No time to study such strategies in detail in this class, but we will discuss some general pitfalls: perhaps X-regional (e.g., X-state) GE relations are fundamentally different from aggregate ones? [E.g. because of openness/trade linkages, or because wage bargaining is national. We will return to this in Lecture Notes 9 & 10.]