Lecture 10: Cross-Sectional Analysis – Aggregation

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14.461, Fall 2022

The aggregation problem

- Last time we characterized the estimand of typical **cross-sectional regressions**
- We saw that there's an issue—the **"aggregation/missing intercept" problem**
    - Estimand of micro experiment is a direct ("partial equilibrium") response, and so free of the general equilibrium effects that enter the full response $\ominus$
      Why missing intercept? looking across agents absorbs common aggregate effects = enters fixed effect in cross-sectional regression on aggregate shock
    - Concrete example: direct response to lump-sum stimulus check misses tax financing, price effects (interest rates, relative goods prices), Keynesian multiplier, ...
- This lecture will discuss **two possible solutions**: [illustrate both for stimulus check application]
    1. Macro as explicitly aggregated micro
    2. Different shocks share identical GE effects (e.g., a common "demand multiplier")

Roadmap

- We'll begin by formalizing the **aggregation problem**
  - Using sequence-space techniques: you can generally write

$$\textbf{total causal effect} = \textbf{micro estimate} + \textbf{GE effects}$$

  - We will illustrate this decomposition in a model of stimulus checks. Will be straightforward to see that such decompositions are very general.
- We will then discuss our two **solution strategies**
  - Both will have a similar flavor: use model structure to arrive at results of the form

$$\textbf{total causal effect} = \textbf{micro estimate} + \textbf{something that is (cleanly?) measurable}$$

  - The approaches will differ in how much model structure is imposed/how easy the measurement part is

Outline

1. The Aggregation Problem

2. Solutions
   - Macro As Explicitly Aggregated Micro
   - Commonality in GE: "Demand Equivalence"

3. Class Summary

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A model of stimulus checks [Wolf (2022)]

- The next slide will provide a brief sketch of a **rich structural GE model**
  - Model blocks: heterogeneous households ["HANK"], heterogeneous firms [Khan-Thomas, Ottonello-Winberry], fiscal & monetary policy
  - Solution method: linearization + perfect foresight [= sequence-space]
  - Policy experiment: date-0 stimulus check, financed with future taxes
- In the context of this lecture the **purpose of the model** is twofold:
  1. First: provide an explicit worked-out example of the decomposition

     total causal effect = micro estimate + GE effects

  2. Later: formally justify our two approaches to solving the aggregation problem

A model of stimulus checks [Wolf (2022)]

1. **Households**

$$\max \quad \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_{it},\ell_{it})\right]$$

$$\text{s.t.} \quad c_{it}+b_{it}=w_t\ell_{it}\boldsymbol{e_{it}}+\boldsymbol{\tau_t}+\frac{1+i_{t-1}^b}{1+\pi_t}b_{it-1}+d_t$$

$+$ borr. constraint $\boldsymbol{b_{it}\geq \underline{b}}$ & ℓ_{it} is demand-determined ($=$ **wage-NKPC**)

2. **Production**: $\approx$ canonical heterogeneous-firm model, e.g. see Ottonello-Winberry (2021)

$$\max \quad \mathbb{E}_0\left[\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1+\pi_s}{1+r_{s-1}^b}\right)d_{jt}\right]$$

$$\text{s.t.} \quad d_{jt}=p_t^l y(z_{jt},k_{jt-1},\ell_{jt})-w_t\ell_{jt}-[k_{jt}-(1-\delta)k_{jt-1}]-\text{adj. costs}-b_{jt}+\frac{1+i_{t-1}^b}{1+\pi_t}b_{jt-1}$$

$+$ fin. constraints on $\{b_{jt}^f, d_{jt}^l\}$ & output is demand-determined ($=$ **price-NKPC**)

3. **Government**: spend & tax, set nominal rate (debt & monetary rules)

A PE-GE decomposition

- **Objective**: find aggregate causal effects of a transfer stimulus policy
  - Policy details: transfer path $\boldsymbol{\tau}^x = \boldsymbol{\tau}^x(\varepsilon_\tau)$ sent out to all households, financed with some (uniform) future tax path $\boldsymbol{\tau}^e = \boldsymbol{\tau}^e(\varepsilon_\tau)$, total taxes/transfers are $\boldsymbol{\tau} = \boldsymbol{\tau}^x + \boldsymbol{\tau}^e$
  - Preview: will later also look at gov't spending shock $\boldsymbol{g} = \boldsymbol{g}(\varepsilon_g)$ (+ tax financing)
- Our key tool for formalizing the aggregation problem will be a **PE**-**GE** decomposition of the household consumption-savings decision: [as usual: use sequence-space cons. function]

$$\hat{\mathbf{c}}_\tau \quad \equiv \quad \mathbf{c}(\mathbf{p}_\tau, \boldsymbol{\tau}_\tau^x) - \mathbf{c}(\bar{\mathbf{p}}, \bar{\boldsymbol{\tau}}^x)$$

  - Notation: bars = steady state, hats & subscripts = IRFs, boldface = time paths
  - HH's receive **transfer $\boldsymbol{\tau}^x$** and face **GE feedback $\mathbf{p}$** ($\mathbf{i}_b$, $\boldsymbol{\pi}$, $\mathbf{w}$, tax financing $\boldsymbol{\tau}^e$, ...)

A PE-GE decomposition

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$$\hat{\mathbf{c}}_\tau \quad = \quad \underbrace{\hat{\mathbf{c}}(\bar{\mathbf{p}}, \hat{\boldsymbol{\tau}}^x_\tau)}_{\text{PE impact: } \hat{\mathbf{c}}^{PE}_\tau} \quad + \quad \underbrace{\hat{\mathbf{c}}(\hat{\mathbf{p}}_\tau, \bar{\boldsymbol{\tau}}^x)}_{\text{GE feedback: } \hat{\mathbf{c}}^{GE}_\tau}$$

  - Notation: bars = steady state, hats & subscripts = IRFs, boldface = paths
  - HH's receive **transfer $\boldsymbol{\tau}^x$** and face **GE feedback p** ($\mathbf{i}_b$, $\boldsymbol{\pi}$, $\mathbf{w}$, tax financing $\boldsymbol{\tau}^e$, ...)

Interpreting micro estimands

- We already discussed that **cross-sectional data** can give **"PE" effects**:
  - Let $\varepsilon_{\tau it} = \xi_{\tau it} \times \varepsilon_{\tau t}$ ($\xi_{\tau it}$ is iid), and consider a cross-sectional regression of the form
  $$c_{it+h} = \alpha_i + \delta_t + \beta_{\tau h} \times \varepsilon_{\tau it} + u_{it+h}, \quad h = 0, 1, 2, \dots$$
  - Then the OLS estimand of $\boldsymbol{\beta}_\tau \equiv (\beta_{\tau 0}, \beta_{\tau 1}, \dots)'$ satisfies
  $$\boldsymbol{\beta}_\tau \times \varepsilon_{\tau t} \quad = \quad \int_0^1 \frac{\partial \mathbf{c}_i}{\partial \varepsilon_{\tau 0}} \, di \times \varepsilon_{\tau t} \quad = \quad \widehat{\mathbf{c}}_\tau^{PE}$$
  - Here I'm using heterogeneous exposure to an agg. shock [as in Johnson-Parker-Souleles]. Last time we instead used shocks w/o aggregate effects [notably, lottery wins]. Both give $\widehat{\mathbf{c}}_\tau^{PE}$.
- **Aggregation problem**: how do we go from the estimable $\widehat{\mathbf{c}}_\tau^{PE}$ to $\hat{\mathbf{c}}_\tau$?
  - In general we are missing the GE term $\widehat{\mathbf{c}}_\tau^{GE}$. Can use $\widehat{\mathbf{c}}_\tau^{PE}$ only if the GE term is zero (= units without a direct treatment show no overall response)
  - "Missing intercept": GE effects that are orthogonal to treatment heterogeneity $\varepsilon_{\tau it}$ are necessarily differenced out [when regressing on an aggregate shock]

A more general discussion

- This approach to **PE-GE** decompositions & aggregation is very general
  - Consider some general outcome of interest x. Suppose x is directly affected by a shock ε and GE "prices" p. Assume in sequence-space you can write

$$\boldsymbol{x} = \mathcal{X}(\boldsymbol{\varepsilon}, \boldsymbol{p})$$

  - Then we get the decomposition

$$\widehat{\boldsymbol{x}} \;=\; \underbrace{\mathcal{X}_\varepsilon \times \boldsymbol{\varepsilon}}_{\text{PE impact}} \;+\; \underbrace{\mathcal{X}_p \times \widehat{\boldsymbol{p}}}_{\text{GE feedback}}$$

  - Can easily see how investment subject to tax incentives q fits into this:

$$\widehat{\boldsymbol{i}} \;=\; \mathcal{I}_q \times \widehat{\boldsymbol{q}} \;+\; \mathcal{I}_w \times \widehat{\boldsymbol{w}} + \mathcal{I}_r \times \widehat{\boldsymbol{r}} + \dots$$

- **Limitation**: assumes price-taking behavior

Other examples

Many other **famous cross-sectional studies** face the same problem:

1. Regional vs. aggregate fiscal multipliers
   Nakamura & Steinsson, Chodorow-Reich
2. China shock (regional import competition & employment)
   Autor, Dorn & Hansen
3. Bank lending cuts to firms
   Chodorow-Reich, Herreño
4. Consumption responses to stock market gains
   Chodorow-Reich, Nenov & Simsek
5. ...

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Solutions

What are possible strategies for solving the aggregation problem?

- Standard approach: match micro estimate in **fully specified structural model**, then use that model for aggregation

  Often not easy to see: what features of the model matter for **PE**-**GE** mapping?

- Recently popular **semi-structural** alternative: impose enough structure so that the "missing intercept" (e.g., $\widehat{\mathbf{c}}_\tau^{GE}$) becomes directly measurable

- I'll here review two examples of this, both for the **stimulus check application**:

  a) Consider a model in which the micro estimates at the same time also pin down $\widehat{\mathbf{c}}_\tau^{GE}$ ("macro as explicitly aggregated micro") Auclert, Rognlie & Straub (2018)

  b) Consider a model in which more readily available time series evidence on other shocks simultaneously pins down $\widehat{\mathbf{c}}_\tau^{GE}$ ("commonality in GE") Wolf (2022)

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Aggregation via the IKC

- Our model so far was quite rich: capital, firm-level financial frictions, general monetary rule, partially flexible prices & wages, ...
- Let's now simplify to return to the **IKC model** from last lecture:
  - Adapted to today's notation, the equilibrium path $\widehat{\boldsymbol{c}}_\tau$ solves
$$\mathcal{C}_y \widehat{\boldsymbol{c}}_\tau + \mathcal{C}_\tau (\widehat{\boldsymbol{\tau}}^x + \widehat{\boldsymbol{\tau}}^e) = \widehat{\boldsymbol{c}}_\tau$$
  - Thus we have
$$\begin{aligned} \widehat{\boldsymbol{c}}_\tau^{PE} &= \mathcal{C}_\tau \widehat{\boldsymbol{\tau}}^x \\ \widehat{\boldsymbol{c}}_\tau^{GE} &= [I - \mathcal{C}_y]^{-1} \mathcal{C}_\tau \left(\widehat{\boldsymbol{\tau}}^x + \widehat{\boldsymbol{\tau}}^e \right) - \mathcal{C}_\tau \widehat{\boldsymbol{\tau}}^x \end{aligned}$$
- The direct effect is $\mathcal{C}_\tau \times \widehat{\boldsymbol{\tau}}^x$. But micro data can (in principle) get all of $\mathcal{C}_y / \mathcal{C}_\tau$ and so (through the model structure) the full GE counterfactual

  Interpretation: $\widehat{\boldsymbol{c}}_\tau$ will reflect (i) direct effect, (ii) fiscal financing rule, and (iii) Keynesian GE multiplier

Aside: getting all of $\mathcal{C}_y/\mathcal{C}_\tau$

- Empirical evidence so far: mostly on first column of $\mathcal{C}_y/\mathcal{C}_\tau$

  → Main takeaway: households on average *gradually* spend (small) lump-sum income receipts
  Fagereng-Holm-Natvik (2020)

- What about the rest? limited empirical evidence, but strong theoretical predictions
  See Wolf (2022). Shape obtains in OLG & bond-in-utility. HANK is just slightly different ...

$$\mathcal{C}_\tau \approx \omega \times \begin{pmatrix} 1 & \frac{\theta}{1+\bar{r}} & \left(\frac{\theta}{1+\bar{r}}\right)^2 & \dots \\ \theta & 1 & \frac{\theta}{1+\bar{r}} & \dots \\ \theta^2 & \theta & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Note the $\approx$. In fact MPCs along the main diagonal decline somewhat, reflecting anticipation.

→ Given (i) first column (from data) + (ii) model extrapolation for rest of $\mathcal{C}_\tau/\mathcal{C}_y$ + (iii) as'n of IKC model structure we can recover $\widehat{\boldsymbol{c}}_\tau$ for *any* stimulus check policy

Discussion

+ Reduce aggregation problem to **micro measurement exercise**
    - All macro identifying assumptions are embedded in the model. Conditional on that, we only need PE matrices, estimable from micro data alone
    - Note: micro experiments both give the direct effect and the way to aggregate it
- Approach requires very **strong restrictions on the model**
    - Recall the simplifying assumptions: no capital, no firm borrowing frictions, single asset, single final good, monetary authority that fixes the real rate (and issues real bonds), ...
- Actually the micro measurement exercise is still too hard, thus **still need model**
    - Micro data are enough to get first column of $\mathcal{C}_\tau$ and $\mathcal{C}_y$, then need extrapolation via HA consumption-savings problem for rest of the matrices

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Commonality in GE

- Could we use empirical evidence on the **aggregate effects of shock/policy A** to learn about the **missing GE effects** for some micro study on **policy B**?
  Could be useful if A is somehow easier to analyze than B in the aggregate time series ...

- Less abstractly: the CBO actually uses a variant of this idea for **"demand shocks"**
  - Micro causal variation can tell us how much stimulus checks/bonus depreciation stimulate "demand" (consumer spending/firm investment expenditure)
  - Time series literature has identified aggregate multipliers of gov't spending increases

can we just combine the two?

- Remainder of this lecture: formalize this idea in the general model from before, discuss implementation details, assess generality & limitations [closely following Wolf (2022)]

Identification result

Proposition ("demand equivalence")

Return to the general model from before, and consider a policy $\boldsymbol{\varepsilon_\tau}$*. Suppose that:*

A1 ...

A2 ...

A3 ...

If a fiscal spending policy shock $\boldsymbol{\varepsilon_g}$ *is s.t. (i)* $\widehat{\boldsymbol{g}}_g = \widehat{\boldsymbol{c}}_\tau^{PE}$ *and (ii)* $\widehat{\boldsymbol{\tau}}_g^e = \widehat{\boldsymbol{\tau}}_\tau^e$*, then, to first order,*

$$\widehat{\boldsymbol{c}}_\tau = \underbrace{\widehat{\boldsymbol{c}}_\tau^{PE}}_{\textbf{PE response}} + \underbrace{\widehat{\boldsymbol{c}}_g}_{\textbf{GE feedback}}$$

Identification result

Proposition ("demand equivalence")

Return to the general model from before, and consider a policy $\boldsymbol{\varepsilon_\tau}$. Suppose that:

A1 *Households and gov't consume a* **common final good**.

This assumption has no bite in our model since it only features one good anyway. I'm stating it just to make clear the importance of this restriction.

A2 *Households and gov't borrow and lend at* **identical rates**.

This assumption has no bite in our model since there is only one asset. Again just stating for emphasis.

A3 *There are* **no wealth effects** *in labor supply and/or wages are fully* **rigid**.

If a fiscal spending policy shock $\boldsymbol{\varepsilon_g}$ is s.t. (i) $\widehat{\boldsymbol{g}}_g = \widehat{\boldsymbol{c}}_\tau^{PE}$ and (ii) $\widehat{\boldsymbol{\tau}}_g^e = \widehat{\boldsymbol{\tau}}_\tau^e$, then, to first order,

$$\widehat{\boldsymbol{c}}_\tau = \underbrace{\widehat{\boldsymbol{c}}_\tau^{PE}}_{\textbf{PE response}} + \underbrace{\widehat{\boldsymbol{c}}_g}_{\textbf{GE feedback}}$$

Proof sketch [see Wolf (2021) for details]

- Equilibrium = solution to many mkt-clearing conditions + other restrictions
  [output, gov't budget, asset markets, labor, equity valuation, asset arbitrage, Taylor rule, ...]

$$\underbrace{\mathbf{H}(\mathbf{p};\boldsymbol{\varepsilon})}_{\text{full equilibrium system}} = \mathbf{0}$$

- Proof strategy: identical **excess demand/supply** *in all markets*

A1 **Output**: by property (i), identical demand pressure for common final good

$$\mathbf{c}(\mathbf{p};\boldsymbol{\varepsilon}) + \mathbf{g}(\boldsymbol{\varepsilon}) = \mathbf{y}(\mathbf{p};\boldsymbol{\varepsilon}) - \mathbf{i}(\mathbf{p};\boldsymbol{\varepsilon})$$

A2 **Gov't budget**: by property (ii), identical tax financing for transfers & gov't spending

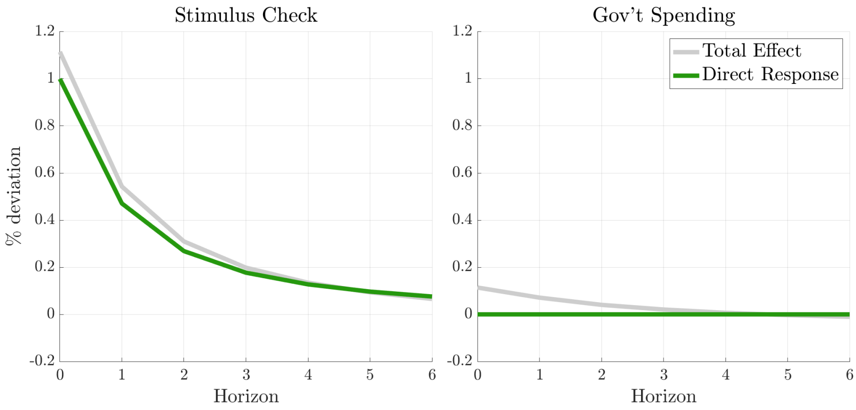
$$\boldsymbol{\tau}^e = \boldsymbol{\tau}^e(\mathbf{p};\boldsymbol{\varepsilon})$$

A3 **Labor**: no shift in labor supply/shift is irrelevant

$$\boldsymbol{\ell}^h(\mathbf{p};\boldsymbol{\varepsilon}) = \boldsymbol{\ell}^f(\mathbf{p};\boldsymbol{\varepsilon})$$

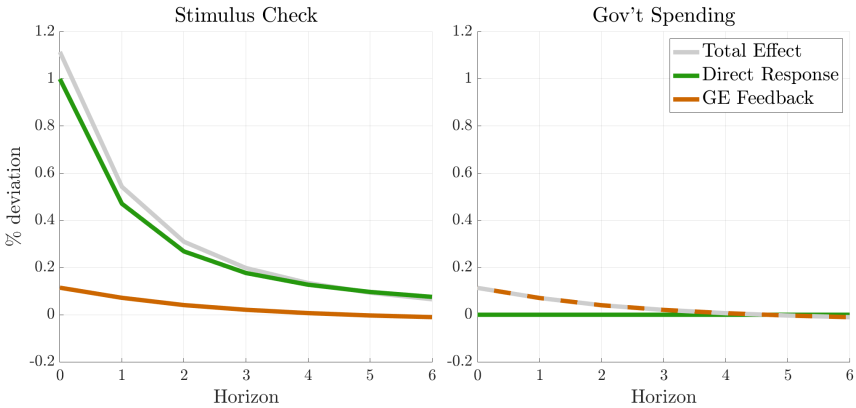
Numerical illustration

Let's now see this result in action in a sticky-wage HANK model ...



Numerical illustration

Let's now see this result in action in a sticky-wage HANK model ...



Wolf

Numerical illustration

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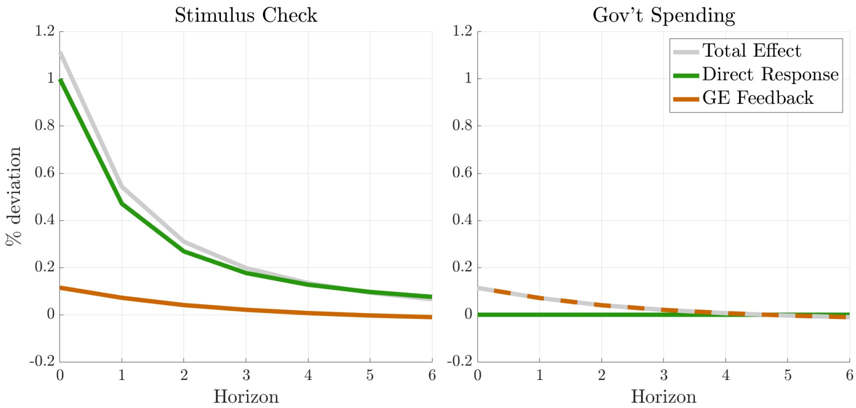


We can leverage this result to operationalize the CBO intuition ...

Measurement inputs

Result suggests that we can combine **cross-sectional** and **time-series** measurement:

1. **Cross-sectional** identification

$$\boldsymbol{\beta}_\tau \times \varepsilon_{\tau t} \quad = \quad \int_0^1 \frac{\partial \mathbf{c}_i}{\partial \varepsilon_{\tau 0}}\, di \times \varepsilon_{\tau t} \quad = \quad \widehat{\mathbf{c}}_\tau^{PE}$$

2. Fiscal **time series** experiments
    - We have seen several time series approaches to identifying fiscal spending shocks and so estimating $\widehat{c}_g$. E.g.: (i) narrative, (ii) forecast errors, (iii) timing/exclusion restrictions Blanchard-Perotti (2002), Mountford-Uhlig (2009), Ramey (2011), Caldara-Kamps (2017), ...
    - Summary: most estimates lie "in a fairly narrow range, 0.6 to 1" Ramey (2018)

$$\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}_{\text{combine them to arrive at } \widehat{c}_\tau}$$

Matching experiments

Challenge: necessary condition to combine experiments is identical net excess demand — $\widehat{\boldsymbol{g}}_g = \widehat{\boldsymbol{c}}_\tau^{PE}$. Given $\widehat{\boldsymbol{c}}_\tau^{PE}$, how can we find the required gov't spending shock?

- Natural approach: look for best **linear combination**
  - Suppose time series analysis has identified several shocks with paths $\widehat{\mathbf{g}}_{g_k}$. Projection:

$$\widehat{\mathbf{c}}_\tau^{PE} \quad = \quad \sum_{k=1}^{n_k} \gamma_k \quad \times \quad \widehat{\mathbf{g}}_{g_k} \quad + \quad \text{error}$$

  - Then

$$\sum_{k=1}^{n_k} \gamma_k \quad \times \quad \widehat{\mathbf{c}}_{g_k}$$

    promises to capture general equilibrium effects *up to the error term*
  - Note that this is the same multi-shock idea as what we used for policy rule counterfactuals

Implementation & interpretation

- Suppose you have ensured that $\widehat{\boldsymbol{g}}_g = \widehat{c}_\tau^{PE}$. Now construct

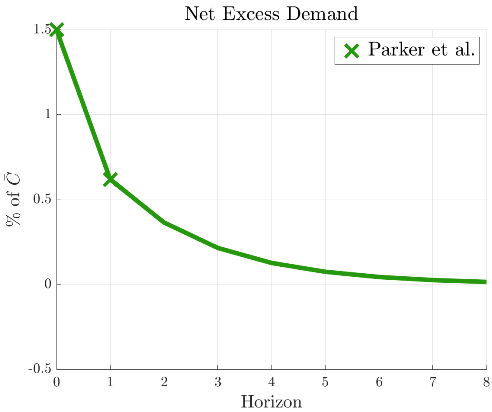
$$\widehat{\mathbf{c}}_\tau = \underbrace{\widehat{\mathbf{c}}_\tau^{PE}}_{\text{PE response}} + \underbrace{\sum_{k=1}^{n_k} \gamma_k \times \widehat{\mathbf{c}}_{g_k}}_{\text{GE feedback}} \tag{1}$$

How should we interpret (1)? Valid **GE** counterfactual for stimulus check shock ε_τ s.t.:

a) ε_τ and ε_g are associated with the same movements in taxes (recall: condition (ii) in th'm)
E.g. for stimulus check policy: same financing rule

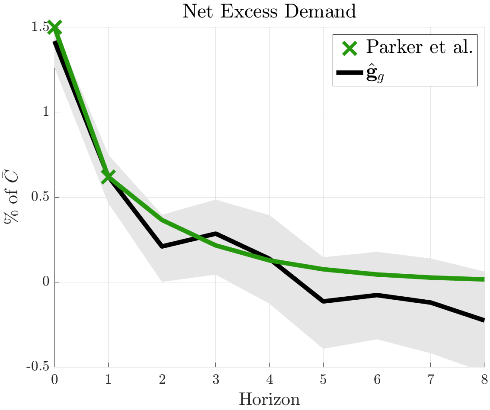
b) ε_τ and ε_g occur in the same macro environment (same policy regime, cyclical state, ...)
Why? recall that th'm used (log-)linearization

Application: PE demand



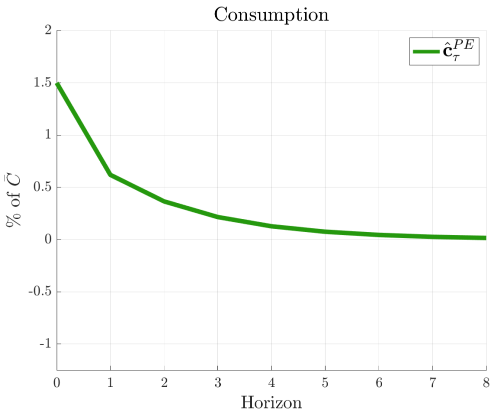
- Cross-sectional identification
  [Parker-Souleles-Johnson-McClelland (2013)]
  - Experiment: one-off stimulus checks (around $600 per household)
  - Find: strong, very short-lived response (here: extrapolated from $t = 2$ onwards)

Application: PE demand



- Cross-sectional identification [Parker-Souleles-Johnson-McClelland (2013)]
  - Experiment: one-off stimulus checks (around $600 per household)
  - Find: strong, very short-lived response (here: extrapolated from $t = 2$ onwards)
- Time series identification [Ramey (2011)]
  - Identification as'n: forecast errors as IV
  - Find: short-lived uptick in g, deficit-financed, muted interest rate response

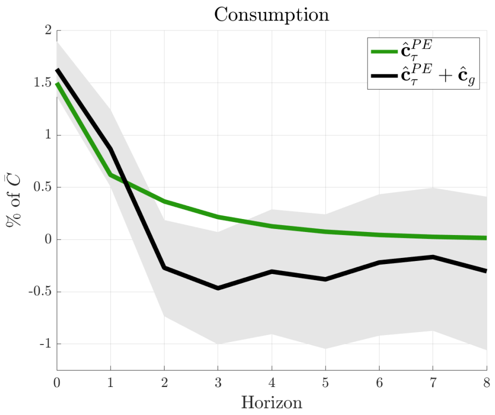
Application: GE counterfactual



- Aggregate via demand equivalence:

$$\widehat{\mathbf{c}}_\tau = \underbrace{\widehat{\mathbf{c}}_d^{PE}}_{\text{PE response}} + \underbrace{\widehat{\mathbf{c}}_g}_{\text{GE feedback}}$$

Application: GE counterfactual



- Aggregate via demand equivalence:

$$\widehat{\mathbf{c}}_\tau = \underbrace{\widehat{\mathbf{c}}_d^{PE}}_{\text{PE response}} + \underbrace{\widehat{\mathbf{c}}_g}_{\text{GE feedback}}$$

- Main result: strong impact stimulus with

  full C response $\approx$ **micro estimate**

- Interpretation: same conclusion in *any* model s.t.:

  (i) demand equivalence holds

  (ii) micro & macro moments are matched

Assessment

How should we interpret these results?

- So far: valid counterfactuals for *any model* that satisfies **demand equivalence**
  demand equivalence + cross-sectional & time series moments from literature $\Rightarrow$ our results (= "sufficient statistics" in public finance)

- But the required assumptions are of course quite **strong**

- **Q**: what does it look like in models that break exact demand equivalence?
  - **A**: probably miss some GE crowding-out = upward-biased ▸ Details

Discussion

+ Applies for a large **space of models**
    - Allows for investment (+ very general firm block) & general monetary rule, though still requires single good & no borrowing frictions
+ Implementable purely through **empirical measurement**
- **Not always applicable**
    - Need to find experiments so that net excess demand paths are aligned
    - Works only for demand shocks, so not a general-purpose aggregation methodology
- Need to **trust time series evidence** for a different shock
    - Requires relatively greater confidence in time series estimates for fiscal spending experiment than the demand shock of interest

Appendix

Demand equivalence: accuracy

- Idea: in models that **break equivalence** compute the plim of aggregation procedure
  1. Estimated HANK model [Smets-Wouters (2007) + Kaplan-Moll-Violante (2018)]
     Note: breaks equivalence *only* via labor assumption
  2. Further extensions that jointly break asn's 1-3
- **Main result**: miss some **GE crowding-out**

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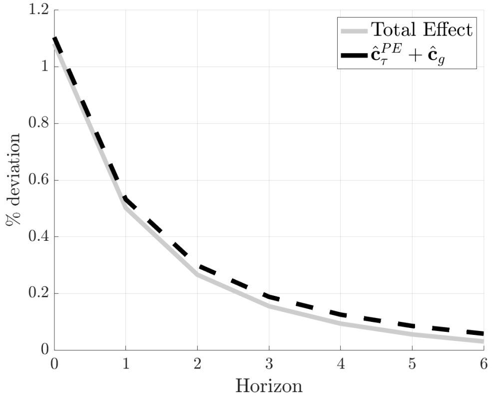
Estimated HANK Model



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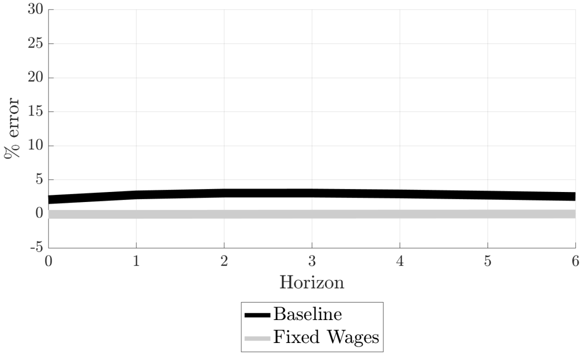
- Estimate HANK model, solve at posterior mode
  Results similar across the posterior distribution Details
- First finding: nearly exact in rich estimated model
- What's going on?
  - Equivalence fails *only* due to labor channel

$$\hat{\mathbf{c}}_\tau = \hat{\mathbf{c}}_\tau^{PE} + \hat{\mathbf{c}}_g + \text{error}\left(\hat{\boldsymbol{\ell}}_\tau^{PE}\right)$$

  - But wages are quite sticky ($\phi_w = 0.6$)
  - Well-known: for transitory fluctuations, with sticky prices/wages, labor wedge shocks matter little
    Christiano (2011, 2012), Auclert et al. (2020)

Further extensions



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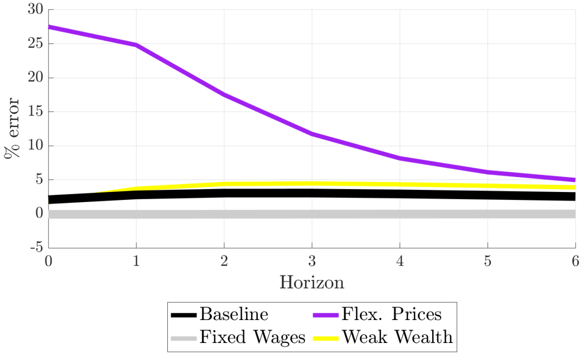
- For several further model variants compute

$$\text{error} = \frac{(c_\tau^{PE} + c_g) - c_\tau}{c_{\tau,0}}$$

- Check violations of each key assumption:

Further extensions



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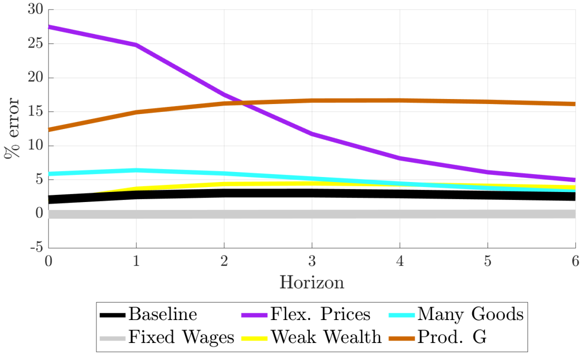
- For several further model variants compute

$$\text{error} \quad = \quad \frac{\left(c_\tau^{PE} + c_g\right) - c_\tau}{c_{\tau,0}}$$

- Check violations of each key assumption:

  A1 flexible wages & alternative preferences

Further extensions



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- For several further model variants compute

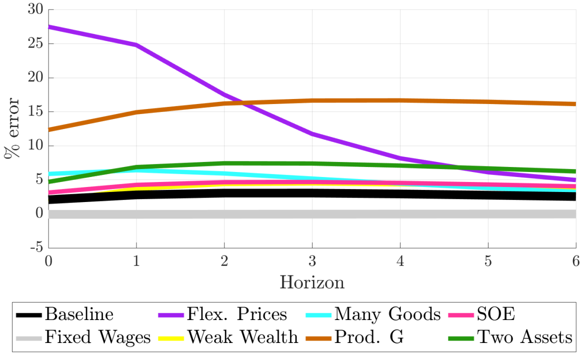
$$\text{error} = \frac{(c_\tau^{PE} + c_g) - c_\tau}{c_{\tau,0}}$$

- Check violations of each key assumption:

  A1 flexible wages & alternative preferences

  A2 multiple goods, gov't investment

Further extensions



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- For several further model variants compute

$$\text{error} \quad = \quad \frac{(c_\tau^{PE} + c_g) - c_\tau}{c_{\tau,0}}$$

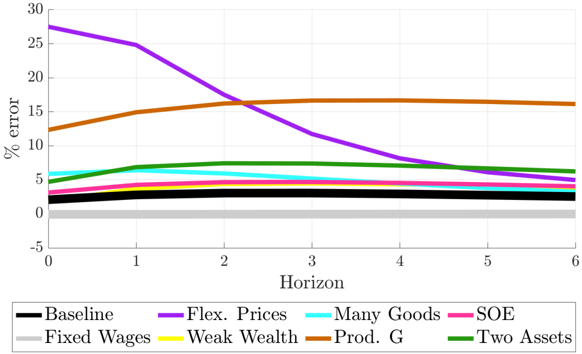
- Check violations of each key assumption:

  A1 flexible wages & alternative preferences

  A2 multiple goods, gov't investment

  A3 multiple assets, small open economy

Further extensions



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- For several further model variants compute

$$\text{error} = \frac{(c_\tau^{PE} + c_g) - c_\tau}{c_{\tau,0}}$$

- Check violations of each key assumption:

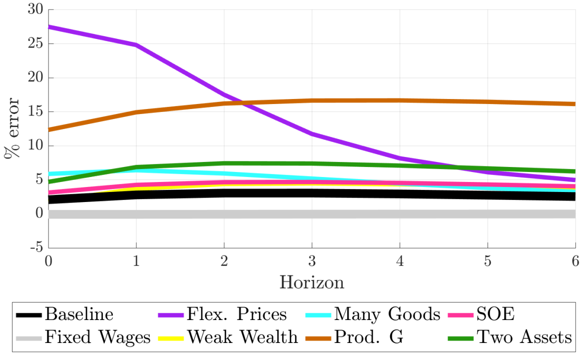
A1 flexible wages & alternative preferences

A2 multiple goods, gov't investment

A3 multiple assets, small open economy

main message: approximation is biased *up*

Further extensions



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- Check violations of each key assumption:

  A1 flexible wages & alternative preferences

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  main message: approximation is biased *up*

- But: bound likely to be tight for checks

  small & transitory, little evidence of labor adjustment, do not use gov't investment, ...

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